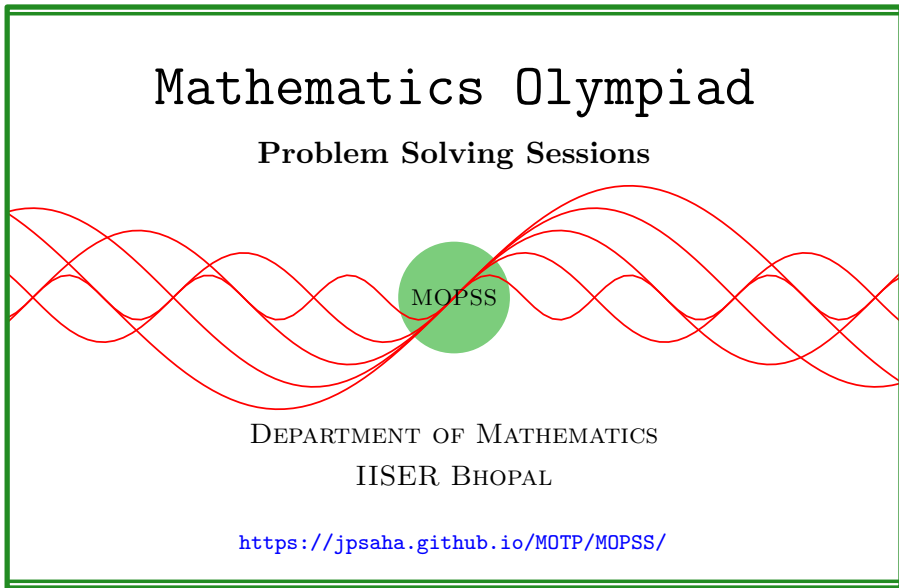


Polygons

MOPSS



Suggested readings

- Evan Chen's advice [On reading solutions](https://blog.evanchen.cc/2017/03/06/on-reading-solutions/), available at <https://blog.evanchen.cc/2017/03/06/on-reading-solutions/>.
- Evan Chen's [Advice for writing proofs/Remarks on English](https://web.evanchen.cc/handouts/english/english.pdf), available at <https://web.evanchen.cc/handouts/english/english.pdf>.
- [Notes on proofs](#) by Evan Chen from [OTIS Excerpts](#) [[Che25](#), Chapter 1].
- [Tips for writing up solutions](https://www.math.utoronto.ca/barbeau/writingup.pdf) by Edward Barbeau, available at <https://www.math.utoronto.ca/barbeau/writingup.pdf>.
- Evan Chen discusses why [math olympiads](#) are a valuable experience for [high schoolers](#) in the post on [Lessons from math olympiads](#), available at <https://blog.evanchen.cc/2018/01/05/lessons-from-math-olympiads/>.

List of problems and examples

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§1 Polygons

Exercise 1.1 (IOQM 2023 P6, AoPS). Let X be the set of all even positive integers n such that the measure of the angle subtended by a side at the center of some regular polygon is n degrees. Find the number of elements in X .

Summary — Determine the positive integers $r \geq 3$ such that dividing 360 by r yields an even number.

Walkthrough —

- (a) Let $r \geq 3$ be an integer and consider a regular r -gon. Note that $360/r$ is even if and only if r is equal to one of

$$3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 24, 36, 45, 60, 120, 360.$$

In other words, there are precisely 16 positive integers $r \geq 3$ such that dividing 360 by r yields an even number.

- (b) It follows that the number of elements of X is equal to 16.

Solution 1. Note that the map

$$r \mapsto \frac{360}{r}$$

is a bijection from the set of positive integers $r \geq 3$ such that $360/r$ is even to the set X , consisting of the even positive integers n such that the measure of the angle subtended by a side at the center of some regular polygon is n degrees. For a positive integer r , note that $360/r$ is even if and only if r divides 180. Using the prime power factorization

$$180 = 2^2 \cdot 3^2 \cdot 5,$$

we see that the number of positive divisors of 180 is

$$(2+1)(2+1)(1+1) = 18.$$

Two of these divisors are 1 and 2, which are less than 3. Thus, the number of positive integers $r \geq 3$ such that $360/r$ is even is

$$18 - 2 = 16.$$

It follows that the number of elements in X is 16. ■

References

- [Che25] EVAN CHEN. *The OTIS Excerpts*. Available at <https://web.evanchen.cc/excerpts.html>. 2025, pp. vi+289 (cited p. 1)