Number Theory

Regional Mathematical Olympiad Problems

1990–2025

§1 Induction

Exercise 1 (RMO 1993 P2,). Prove that the ten's digit of any power of 3 is even.

Exercise 2 (RMO 2025a P6,). Define the sequence $\langle a_0, a_1, a_2, \dots \rangle$ as follows: $a_0 = 49$ and $a_n = 10^{2^n} a_{n-1} - 1$ for $n \geq 1$. Show that $s(a_n^2) = n^2 + n + 7$ for all $n \geq 0$, where $s(m)$ denotes the sum of digits in base 10 representation of a nonnegative integer m.

§2 Quadratic polynomials

Exercise 3 (RMO 1991 P6,). Find all integer values of a such that the quadratic expression

$$(x + a)(x + 1991) + 1$$

can be factored as a product $(x + b)(x + c)$ where b and c are integers.

Exercise 4 (RMO 1999 P7,). Find the number of quadratic polynomials, $ax^2 + bx + c$, which satisfy the following conditions:

1. a, b, c are distinct,
2. $a, b, c \in \{1, 2, 3, \dots, 1999\}$ and
3. $x + 1$ divides $ax^2 + bx + c$.

Exercise 5 (RMO 2013a P6,). Suppose that m and n are integers such that both the quadratic equations $x^2 + mx - n = 0$ and $x^2 - mx + n = 0$ have integer roots. Prove that n is divisible by 6.

§3 Cubic polynomials

Exercise 6 (RMO 2000 P2,). Solve the equation $y^3 = x^3 + 8x^2 - 6x + 8$, for positive integers x and y.

Exercise 7 (RMO 2015f P5). Solve the equation $y^3 + 3y^2 + 3y = x^3 + 5x^2 - 19x + 20$ for positive integers x, y.

§4 Using identities

Exercise 8 (RMO 2004 P3,). Let α and β be the roots of the quadratic equation $x^2 + mx - 1 = 0$, where m is an odd integer. Let $\lambda_n = \alpha^n + \beta^n$, for $n \geq 0$. Prove that for $n \geq 0$,

1. λ_n is an integer, and
2. $\gcd(\lambda_n, \lambda_{n+1}) = 1$.

Exercise 9 (RMO 2006 P6,). Prove that there are infinitely many positive integers n such that $n(n+1)$ can be expressed as a sum of two positive squares in at least two different ways. (Here $a^2 + b^2$ and $b^2 + a^2$ are considered as the same representation.)

§5 Binomial coefficients

Exercise 10 (RMO 1992 P3,). Determine the largest 3-digit prime factor of the integer $\binom{2000}{1000}$.

§6 Congruences

Exercise 11 (RMO 1990 P4,). Find the remainder when 2^{1990} is divided by 1990.

Exercise 12 (RMO 1990 P6,). N is a 50 digit number (in the decimal scale). All digits except the 26th digit (from the left) are 1. If N is divisible by 13, find the 26th digit.

Exercise 13 (RMO 1991 P3,). A four-digit number has the following properties:

1. it is a perfect square,
2. its first two digits are equal to each other,
3. its last two digits are equal to each other.

Find all such four-digit numbers.

Exercise 14 (RMO 1991 P7, , cf. All-Russian MO 1989 Grade 8 Day 2 P5 **??**, India BStat 2006 P3)**.** [**Eng98**, p. 121] Show that $n^4 + 4^n$ is prime if and only if $n = 1$.

Exercise 15 (RMO 1992 P2, , cf. Moscow Mathematical Olympiad 1973 Grade 8 First Round P4)**.** If $\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$, where a, b, c are positive integers with no common factor, prove that $(a + b)$ is the square of an integer.

Exercise 16 (RMO 1993 P5,)**.** Show that $19^{93} - 13^{99}$ is a positive integer divisible by 162.

Exercise 17 (RMO 1994 P3,)**.** Find all 6-digit natural numbers $a_1a_2a_3a_4a_5a_6$ formed by using the digits $1, 2, 3, 4, 5, 6$, once each such that the number $a_1a_2 \ldots a_k$ is divisible by k, for $1 \leq k \leq 6$.

Exercise 18 (RMO 1994 P5,)**.** Let A be a set of 16 positive integers with the property that the product of any two distinct numbers of A will not exceed 1994. Show that there are two numbers a and b in A which are not relatively prime.

Exercise 19 (RMO 1995 P2,)**.** Call a positive integer n *good* if there are n integers, positive or negative, and not necessarily distinct, such that their sum and products are both equal to n. Show that the integers of the form $4k + 1$ and 4ℓ are good.

Exercise 20 (RMO 1995 P3, , cf. Tournament of Towns)**.** [**Tao06**, Problem 2.1] Prove that among any 18 consecutive three digit numbers there is at least one number which is divisible by the sum of its digits.

Exercise 21 (RMO 1996 P4,)**.** Suppose N is an n-digit positive integer such that

1. all the n-digits are distinct, and
2. the sum of any three consecutive digits is divisible by 5.

Prove that n is at most 6. Further, show that starting with any digit one can find a six-digit number with these properties.

Exercise 22 (RMO 1997 P2,)**.** For each positive integer n, define $a_n = 20 + n^2$, and $d_n = \gcd(a_n, a_{n+1})$. Find the set of all values that are taken by d_n and show by examples that each of these values are attained.

Exercise 23 (RMO 1998 P2,)**.** Let n be a positive integer and $p_1, p_2, \ldots, p_n$ be n prime numbers all larger than 5 such that 6 divides $p_1^2 + p_2^2 + \cdots + p_n^2$. Prove that 6 divides n.

Exercise 24 (RMO 1998 P5,). Find the minimum possible least common multiple (lcm) of twenty (not necessarily distinct) natural numbers whose sum is 801.

Exercise 25 (RMO 1999 P2,). Find the number of positive integers which divide 10^{999} but not 10^{998}.

Exercise 26 (RMO 2000 P6,).

1. Consider two positive integers a and b which are such that $a^a b^b$ is divisible by 2000. What is the least possible value of the product ab?
2. Consider two positive integers a and b which are such that $a^b b^a$ is divisible by 2000. What is the least possible value of the product ab?

Exercise 27 (RMO 2001 P7,). Prove that the product of the first 1000 positive even integers differs from the product of the first 1000 odd integers by a multiple of 2001.

Exercise 28 (RMO 2002 P3, , cf. RMO 2012d P2, RMO 2012a P2, RMO 2012b P2, RMO 2012c P2). Let a, b, c be positive integers such that a divides b^2, b divides c^2 and c divides a^2. Prove that abc divides $(a+b+c)^7$.

Exercise 29 (RMO 2003 P2,). If n is an integer greater than 7, prove that $\binom{n}{7} - \left[\frac{n}{7}\right]$ is divisible by 7. [Here $\binom{n}{7}$ denotes the number of ways of choosing 7 objects from among n objects; also for any real number x, $[x]$ denotes the greatest integer not exceeding x.]

Exercise 30 (RMO 2004 P6, , Australian MO 1982). Let $(p_1, p_2, p_3, \ldots, p_n, \ldots)$ be a sequence of primes, defined by $p_1 = 2$ and for $n \geq 1$, p_{n+1} is the largest prime factor of $p_1 p_2 \cdots p_n + 1$. Prove that $p_n \neq 5$ for any n.

Exercise 31 (RMO 2005 P2,). If x, y are integers and 17 divides both the expressions $x^2 - 2xy + y^2 - 5x + 7y$ and $x^2 - 3xy + 2y^2 + x - y$, then prove that 17 divides $xy - 12x + 15y$.

Exercise 32 (RMO 2006 P2,). Find the least possible value of $a + b$, where a, b are positive integers such that 11 divides $a + 13b$ and 13 divides $a + 11b$.

Exercise 33 (RMO 2009 P3,). Show that $3^{2008} + 4^{2009}$ can be written as product of two positive integers each of which is larger than 2009^{182}.

Exercise 34 (RMO 2011a P3,). A natural number n is chosen strictly between two consecutive perfect squares. The smaller of these two squares is obtained by subtracting k from n and the larger by adding ℓ to n. Prove that $n - k\ell$ is a perfect square.

Exercise 35 (RMO 2011b P2, , All-Russian MO 1992–1993 Grade 9 Final Stage P1). Let n be a positive integer such that $2n + 1$ and $3n + 1$ are both perfect squares. Show that $5n + 3$ is a composite number.

Exercise 36 (RMO 2012a P2, cf. RMO 2002 P3, RMO 2012d P2, RMO 2012b P2, RMO 2012c P2)**.** Let a, b, c be positive integers such that $a|b^3, b|c^3$ and $c|a^3$. Prove that abc divides $(a+b+c)^{13}$.

Exercise 37 (RMO 2012b P2, cf. RMO 2002 P3, RMO 2012d P2, RMO 2012a P2, RMO 2012c P2)**.** Let a, b, c be positive integers such that $a|b^4, b|c^4$ and $c|a^4$. Prove that abc divides $(a+b+c)^{21}$.

Exercise 38 (RMO 2012c P2, cf. RMO 2002 P3, RMO 2012d P2, RMO 2012a P2, RMO 2012b P2)**.** Let a, b, c be positive integers such that $a|b^5, b|c^5$ and $c|a^5$. Prove that abc divides $(a+b+c)^{31}$.

Exercise 39 (RMO 2012d P2, cf. RMO 2002 P3, RMO 2012a P2, RMO 2012b P2, RMO 2012c P2)**.** Let a, b, c be positive integers such that a divides b^2, b divides c^2 and c divides a^2. Prove that abc divides $(a+b+c)^7$.

Exercise 40 (RMO 2012e P6)**.** A computer program generated 175 positive integers at random, none of which had a prime divisor greater than 10. Prove that there are three numbers among them whose product is the cube of an integer.

Exercise 41 (RMO 2012f P2)**.** Let n be a positive integer such that 13 divides $n^2 + 3n + 51$. Show that 169 divides $21n^2 + 89n + 44$.

Exercise 42 (RMO 2013d P2)**.** Determine the smallest prime that does not divide any five-digit number whose digits are in a strictly increasing order.

Exercise 43 (RMO 2013e P5)**.** Let a_1, b_1, c_1 be natural numbers. We define

$$a_2 = \gcd(b_1, c_1), \quad b_2 = \gcd(c_1, a_1), \quad c_2 = \gcd(a_1, b_1)$$

and

$$a_3 = \operatorname{lcm}(b_2, c_2), \quad b_3 = \operatorname{lcm}(c_2, a_2), \quad c_3 = \operatorname{lcm}(a_2, b_2).$$

Show that $\gcd(b_3, c_3) = a_2$.

Exercise 44 (RMO 2014b P3)**.** Find all pairs of (x, y) of positive integers such that $2x + 7y$ divides $7x + 2y$.

Exercise 45 (RMO 2014b P4)**.** For any positive integer $n > 1$, let $P(n)$ denote the largest prime not exceeding n. Let $N(n)$ denote the next prime larger than $P(n)$. (For example $P(10) = 7$ and $N(10) = 11$, while $P(11) = 11$ and $N(11) = 13$.) If $n + 1$ is a prime number, prove that the value of the sum

$$\frac{1}{P(2)N(2)} + \frac{1}{P(3)N(3)} + \frac{1}{P(4)N(4)} + \cdots + \frac{1}{P(n)N(n)} = \frac{n-1}{2n+2}.$$

Exercise 46 (RMO 2014c P3)**.** Prove that for any natural number $n < 2310$, $n(2310 - n)$ is not divisible by 2310.

Exercise 47 (RMO 2014d P3,). Determine all pairs $m > n$ of positive integers such that

$$1 = \gcd(n+1, m+1) = \gcd(n+2, m+2) = \cdots = \gcd(m, 2m-n).$$

Exercise 48 (RMO 2015f P3). Let

$$N = 2^5 + 2^{5^2} + 2^{5^3} + \cdots + 2^{5^{2015}},$$

written in the usual decimal form, find the last two digits of the number N.

Exercise 49 (RMO 2016c P3,). Let a, b, c, d, e, f be positive integers such that

$$\frac{a}{b} < \frac{c}{d} < \frac{e}{f}.$$

Suppose $af - be = -1$. Show that $d \geq b + f$.

Exercise 50 (RMO 2016d P3,). The present ages in years of two brothers A and B, and their father C are three distinct positive integers a, b, and c respectively. Suppose $\frac{b-1}{a-1}$ and $\frac{b+1}{a+1}$ are two consecutive integers, and $\frac{c-1}{b-1}$ and $\frac{c+1}{b+1}$ are two consecutive integers. If $a + b + c \leq 150$ determine a, b and c.

Exercise 51 (RMO 2016e P2,). Consider a sequence $(a_k)_{k\geq 1}$ of natural numbers defined as follows: $a_1 = a$ and $a_2 = b$ with $a, b > 1$ and $\gcd(a, b) = 1$ and for all $k > 0$, $a_{k+2} = a_{k+1} + a_k$. Prove that for all natural numbers n and k, $\gcd(a_n, a_{n+k}) < \frac{a_k}{2}$.

Exercise 52 (RMO 2016g P3,). a, b, c, d are integers such that $ad + bc$ divides each of a, b, c and d. Prove that $ad + bc = \pm 1$.

Exercise 53 (RMO 2016g P6,). Positive integers a, b, c satisfy $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} < 1$. Prove that $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \leq \frac{41}{42}$. Also prove that equality in fact holds in the second inequality.

Exercise 54 (RMO 2017a P2,). Show that the equation

$$a^3 + (a+1)^3 + \cdots + (a+6)^3 = b^4 + (b+1)^4$$

has no solutions in integers a, b.

Exercise 55 (RMO 2017b P2). For any positive integer n, let $d(n)$ denote the number of positive divisors of n; and let $\varphi(n)$ denote the number of elements from the set $\{1, 2, \ldots, n\}$ that are coprime to n. (For example, $d(12) = 6$ and $\varphi(12) = 4$.) Find the smallest positive integer n such that $d(\varphi(n)) = 2017$.

Exercise 56 (RMO 2018a P3,). For a rational number r, its *period* is the length of the smallest repeating block in its decimal expansion, for example, the number $r = 0.123123123\ldots$ has period 3. If S denotes the set of all rational numbers of the form $r = 0.\overline{abcdefgh}$ having period 8, find the sum of all elements in S.

Exercise 57 (RMO 2018a P5,). Find all natural numbers n such that $1 + [\sqrt{2n}]$ divides $2n$. (For any real number x, $[x]$ denotes the largest integer not exceeding x.)

Exercise 58 (RMO 2019b P1,). For each $n \in \mathbb{N}$ let d_n denote the greatest common divisor of n and $(2019 - n)$. Find value of $d_1 + d_2 + \cdots + d_{2018} + d_{2019}$.

Exercise 59 (RMO 2023a P2,). Given a prime number p such that $2p$ is equal to the sum of the squares of some four consecutive positive integers. Prove that $p - 7$ is divisible by 36.

Exercise 60 (RMO 2023b P1,). Let $\mathbb{N}$ be the set of all positive integers and

$$S = \left\{(a, b, c, d) \in \mathbb{N}^4 : a^2 + b^2 + c^2 = d^2\right\}.$$

Find the largest positive integer m such that m divides $abcd$ for all $(a, b, c, d) \in S$.

Exercise 61 (RMO 2024a P1,). Find all positive integers x, y such that $202x + 4x^2 = y^2$.

Exercise 62 (RMO 2024a P4,). Let $n > 1$ be a positive integer. Call a rearrangement $a_1, a_2, \ldots, a_n$ of $1, 2, \ldots, n$ *nice* if for every $k = 2, 3, \ldots, n$, we have that $a_1^2 + a_2^2 + \cdots + a_k^2$ is not divisible by k. Determine which positive integers $n > 1$ have a nice arrangement.

Exercise 63 (RMO 2024b P1,). Let $n > 1$ be a positive integer. Call a rearrangement $a_1, a_2, \ldots, a_n$ of $1, 2, \ldots, n$ *nice* if for every $k = 2, 3, \ldots, n$, we have that $a_1 + a_2 + \cdots + a_k$ is not divisible by k.

1. If $n > 1$ is odd, prove that there is no nice arrangement of $1, 2, \ldots, n$.
2. If n is even, find a nice arrangement of $1, 2, \ldots, n$.

Exercise 64 (RMO 2024b P2,). For a positive integer n, let $R(n)$ be the sum of the remainders when n is divided by $1, 2, \ldots, n$. For example, $R(4) = 0+0+1+0 = 1$, $R(7) = 0+1+1+3+2+1+0 = 8$. Find all positive integers such that $R(n) = n - 1$.

Exercise 65 (RMO 2025b P4,). Prove that there do not exist positive rational numbers x and y such that

$$x + y + \frac{1}{x} + \frac{1}{y} = 2025.$$

§7 Systems of equations

Exercise 66 (RMO 1992 P1,). Determine the set of integers n for which $n^2 + 19n + 92$ is a square of an integer.

Exercise 67 (RMO 1995 P4,). Show that the quadratic equation $x^2 + 7x - 14(q^2 + 1) = 0$, where q is an integer, has no integer root.

Exercise 68 (RMO 1996 P2,). Find all triples (a, b, c) of positive integers such that

$$\left(1 + \frac{1}{a}\right)\left(1 + \frac{1}{b}\right)\left(1 + \frac{1}{c}\right) = 3.$$

Exercise 69 (RMO 1999 P6,). Find all solutions in integers m, n of the equation

$$(m - n)^2 = \frac{4mn}{m + n - 1}.$$

Exercise 70 (RMO 2001 P2,). Find all primes p, q such that $p^2 + 7pq + q^2$ is a perfect square.

Exercise 71 (RMO 2005 P6,). Determine all triples (a, b, c) of positive integers such that $a \le b \le c$ and

$$a + b + c + ab + bc + ca = abc + 1.$$

Exercise 72 (RMO 2007 P2,). Let a, b, c be three natural numbers such that $a < b < c$ and $\gcd(c - a, c - b) = 1$. Suppose there exists an integer d such that $a + d, b + d, c + d$ form the sides of a right-angled triangle. Prove that there exist integers ℓ, m such that $c + d = \ell^2 + m^2$.

Exercise 73 (RMO 2008 P2,). Prove that there exist two infinite sequences $\{a_n\}_{n\ge1}$ and $\{b_n\}_{n\ge1}$ of positive integers such that the following conditions hold simultaneously:

(i) $0 < a_1 < a_2 < a_3 < \cdots$,

(ii) $a_n < b_n < a_n^2$ for all $n \ge 1$,

(iii) $a_n - 1$ divides $b_n - 1$ for all $n \ge 1$,

(iv) $a_n^2 - 1$ divides $b_n^2 - 1$ for all $n \ge 1$.

Exercise 74 (RMO 2008 P5,). Three nonzero real numbers a, b, c are said to be in harmonic progression if $\frac{1}{a} + \frac{1}{c} = \frac{2}{b}$. Find all three term harmonic progressions a, b, c of strictly increasing positive integers in which $a = 20$ and b divides c.

Exercise 75 (RMO 2009 P2,). Show that there is no integer a such that $a^2 - 3a - 19$ is divisible by 289.

Exercise 76 (RMO 2010 P4,). Find three distinct positive integers with the least possible sum such that the sum of the reciprocals of any two integers among them is an integral multiple of the reciprocal of the third integer.

Exercise 77 (RMO 2010 P6,). For each integer $n \geq 1$, define

$$a_n = \left[\frac{n}{[\sqrt{n}]} \right],$$

where $[x]$ denotes the largest integer not exceeding x, for any real number x. Find the number of all n in the set $\{1, 2, 3, \ldots, 2010\}$ for which $a_n > a_{n+1}$.

Exercise 78 (RMO 2012c P6,). Find all positive integers such that $3^{2n} + 3n^2 + 7$ is a perfect square.

Exercise 79 (RMO 2012e P3,). Find all natural numbers x, y, z such that

$$(2^x - 1)(2^y - 1) = 2^{2^z} + 1.$$

Exercise 80 (RMO 2012f P5,). Determine all positive integers a, b, c such that $\frac{1}{a} + \frac{2}{b} + \frac{3}{c} = 1$, $a \leq b \leq c$ and a is a prime.

Exercise 81 (RMO 2013a P3,). Find all primes p and q such that p divides $q^2 - 4$ and q divides $p^2 - 1$.

Exercise 82 (RMO 2013a P4,). Find the number of 10-tuples $(a_1, a_2, \ldots, a_9, a_{10})$ of integers such that $|a_1| \leq 1$ and

$$a_1^2 + a_2^2 + a_3^2 + \cdots + a_{10}^2 - a_1a_2 - a_2a_3 - a_3a_4 - \cdots - a_9a_{10} - a_{10}a_1 = 2.$$

Exercise 83 (RMO 2013b P1,). Prove that there do not exist natural numbers x and y with $x > 1$ such that

$$\frac{x^7 - 1}{x - 1} = y^5 + 1.$$

Exercise 84 (RMO 2013c P2,). Find all 4-tuples (a, b, c, d) of natural numbers with $a \leq b \leq c$ and $a! + b! + c! = 3^d$.

Exercise 85 (RMO 2013e P2,). Find all triples (p, q, r) of primes such that $pq = r + 1$ and $2(p^2 + q^2) = r^2 + 1$.

Exercise 86 (RMO 2014c P6,). For any natural number, let $S(n)$ denote sum of digits of n. Find the number of 3 digit numbers for which $S(S(n)) = 2$.

Exercise 87 (RMO 2014e P2,). The roots of the equation

$$x^3 - 3ax^2 + bx + 18c = 0$$

form a non-constant arithmetic progression and the roots of the equation

$$x^3 + bx^2 + x - c^3 = 0$$

form a non-constant geometric progression. Given that a, b, c are real numbers, find all positive integral values a and b.

Exercise 88 (RMO 2015b P3,). Find all integers a, b, c such that $a^2 = bc + 4$ and $b^2 = ca + 4$.

Exercise 89 (RMO 2015b P6,). Find the number of integers m that satisfy both the following properties:

1. $1 \leq m \leq 5000$,
2. $[\sqrt{m}] = [\sqrt{m + 125}]$.

Exercise 90 (RMO 2015c P4,). Find all three digit natural numbers of the form $(abc)_{10}$ such that $(abc)_{10}$, $(bca)_{10}$, $(cab)_{10}$ are in geometric progression. (Here $(abc)_{10}$ is representation in base 10).

Exercise 91 (RMO 2015d P3,). Find all integers a, b, c such that $a^2 = bc + 1$ and $b^2 = ca + 1$.

Exercise 92 (RMO 2016a P3,). For any natural number n, expressed in base 10, let $S(n)$ denote the sum of all digits of n. Find all natural numbers n such that $n = 2S(n)^2$.

Exercise 93 (RMO 2016b P3,). For any natural number n, expressed in base 10, let $S(n)$ denote the sum of all digits of n. Find all natural numbers n such that $n^3 = 8S(n)^3 + 6nS(n) + 1$.

Exercise 94 (RMO 2016f P1,). Find distinct positive integers $n_1 < n_2 < \cdots < n_7$ with the least possible sum, such that their product $n_1 \times n_2 \times \cdots \times n_7$ is divisible by 2016.

Exercise 95 (RMO 2023b P3,). For any natural number n, expressed in base 10, let $s(n)$ denote the sum of all its digits. Find all natural numbers m and n such that $m < n$ and

$$(s(n))^2 = m \text{ and } (s(m))^2 = n.$$

Exercise 96 (RMO 2025a P1,). Solve the following system of equations in nonnegative integers $a_1, a_2, \ldots, a_8$ where $a_i \neq 1$ for $i = 1, \ldots, 8$:

$$\begin{aligned} a_1a_2 &= a_3 + a_4, \\ a_3a_4 &= a_5 + a_6, \\ a_5a_6 &= a_7 + a_8, \\ a_7a_8 &= a_1 + a_2. \end{aligned}$$

§8 Chinese remainder theorem

Exercise 97 (RMO 2015a P3, , Canadian Mathematical Olympiad 1991 P1, ??). Show that there are infinitely many triples (x, y, z) of integers, such that $x^3 + y^4 = z^{31}$.

Exercise 98 (RMO 2018b P3,). Show that there are infinitely many tuples (a, b, c, d) of natural numbers such that $a^3 + b^4 + c^5 = d^7$.

§9 Order

Exercise 99 (RMO 2014a P3, , cf. Tournament of Towns, Fall 1988, Training)**.** [**Tao06**, Problem 2.2] [**AE11**, Problem 3.81] Suppose for some positive integers r and s, the integer 2^r is obtained by permuting the digits of 2^s in decimal expansion and $2^r, 2^s$ have same number of digits. Prove that $r = s$.

§10 Multiplicative functions

Exercise 100 (RMO 2019b P1,)**.** For each $n \in \mathbb{N}$ let d_n denote the greatest common divisor of n and $(2019 - n)$. Find value of $d_1 + d_2 + \cdots + d_{2018} + d_{2019}$.

§11 Zsigmondy's theorem

Exercise 101 (RMO 2012e P3,)**.** Find all natural numbers x, y, z such that

$$(2^x - 1)(2^y - 1) = 2^{2^z} + 1.$$

References

[**AE11**] Titu Andreescu and Bogdan Enescu. *Mathematical Olympiad treasures*. Second. Birkhäuser/Springer, New York, 2011, pp. viii+253. ISBN: 978-0-8176-8252-1; 978-0-8176-8253-8 (cited p. 11)

[**Eng98**] Arthur Engel. *Problem-solving strategies*. Problem Books in Mathematics. Springer-Verlag, New York, 1998, pp. x+403. ISBN: 0-387-98219-1 (cited p. 3)

[**Tao06**] Terence Tao. *Solving mathematical problems*. A personal perspective. Oxford University Press, Oxford, 2006, pp. xii+103. ISBN: 978-0-19-920560-8; 0-19-920560-4 (cited pp. 3, 11)