Orthocentre and Circumcentre

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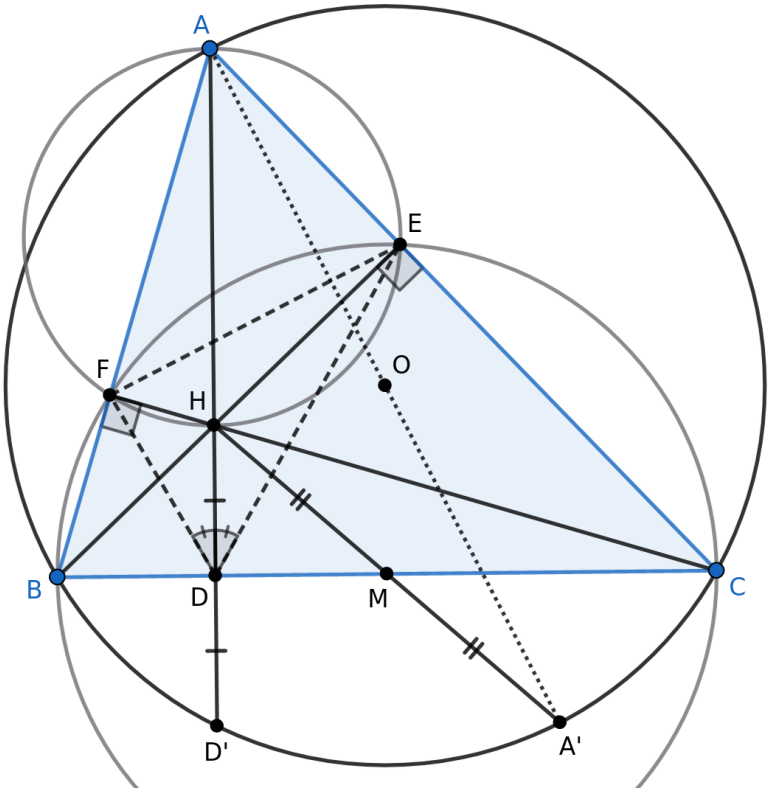

1 Properties

In ΔABC, AD, BE, CF : altitudes, H : orthocentre, O : circumcentre, M : midpoint of BC

(i) $\angle BAH = 90 - B$, $\angle CAH = 90 - C$, $\angle BHC = 180 - A$

(ii) $\angle BAO = 90 - C$, $\angle CAO = 90 - B$, $\angle BOC = 2A$

(iii) Quadrialterals $AEHF$,$BFHD$,$CDHE$ are cyclic.

(iv) Quadrialterals $BEFC$,$CDFA$, $AEDB$ are cyclic.

(v) H is the incentre of $\triangle DEF$.

(vi) Reflection of H in BC lies on (ABC).

(vii) Reflection of H in M lies on (ABC), it is also diametrically opposite to A.

Problem 1. *ME and MF are tangents to (AEF).*

Problem 2. *Let X be the foot of perpendicular from H on AM. Then B, H, C, X are cyclic.*

Nine-point-centre

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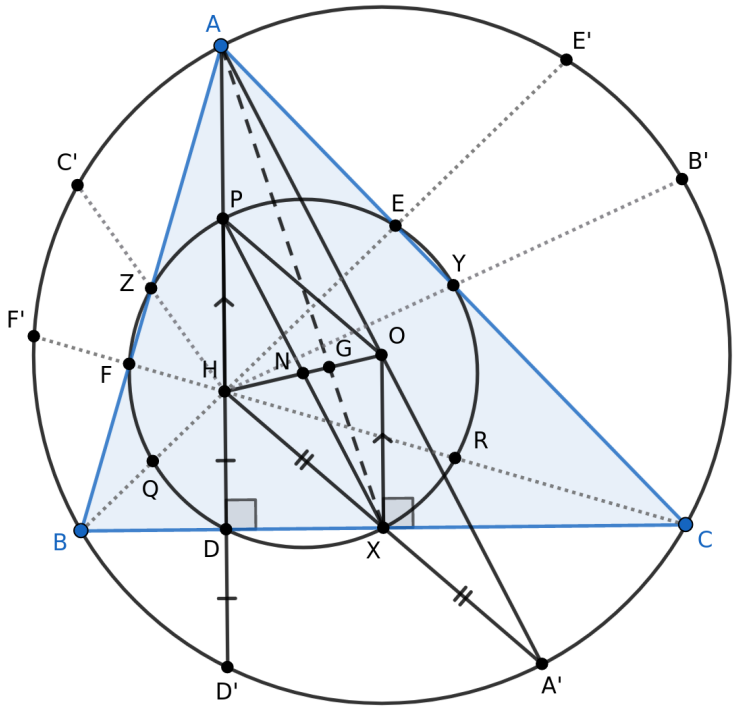

Properties

In ΔABC, N : nine-point-centre, G : centroid, X : midpoint of BC, P : midpoint of AH

(i) $P \in (DEF)$ similarly Q, R

(ii) $PRXD$ is cyclic $\implies$ $X \in (DEF)$ similarly Y, Z

(iii) $D, E, F, P, Q, R, X, Y, Z$ lie on a circle called as nine-point-circle

(iv) it's centre N (nine-point-centre) is midpoint of OH

(v) it's radius is half the circumradius ($NP = \frac{1}{2}OA$)

(vi) $\frac{1}{2}AH = AP = PH = OX$, $PHXO$ and $APXO$ are $\|^{gm}$

(vii) $[ABG] = [BCG] = [CAG]$, $AG = 2GX$

(viii) $G \in OH$, $HG = 2GO$

Problem 3. *For any point T on (ABC), midpoint of TH lies on (DEF).*

Problem 4. *Points P, G and A' are collinear.*

Incentre and Excentre

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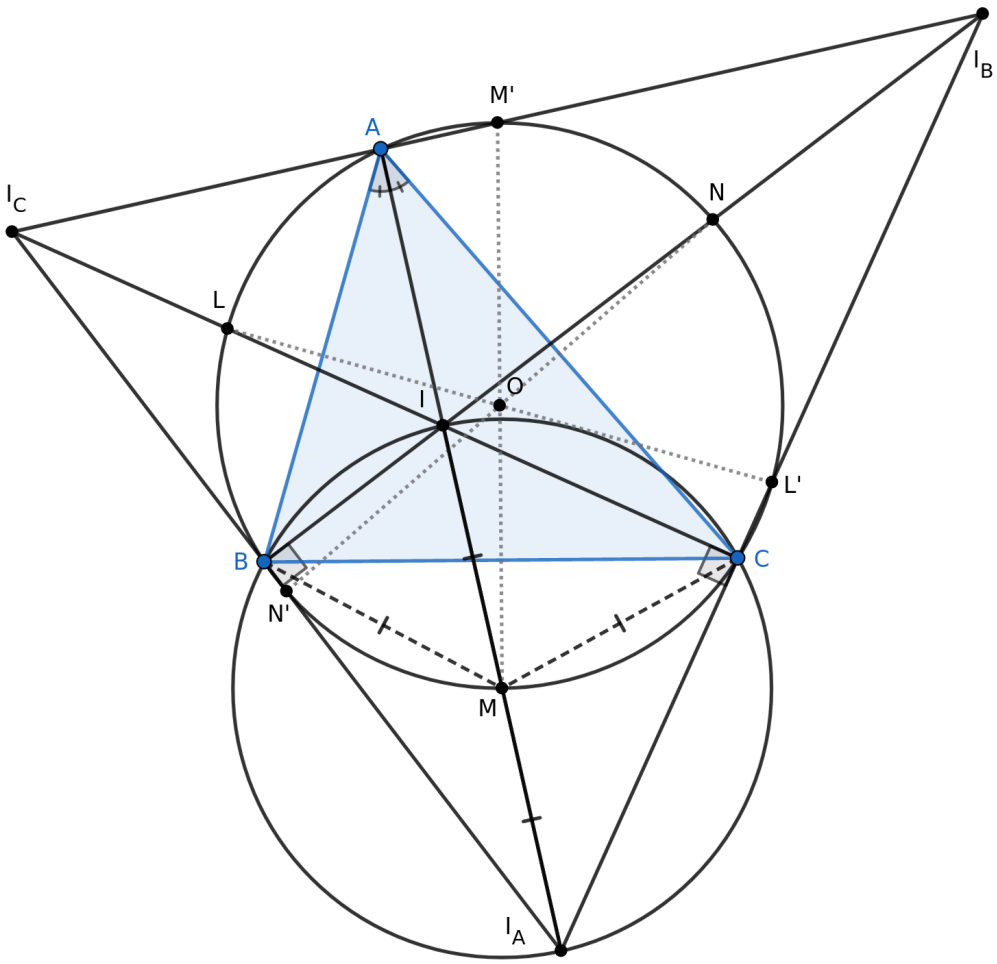

Properties

In ΔABC, M : midpoint of $\widehat{BC}$, I : Incentre, I_A : A-excentre

(i) $\angle BIC = 90 + \frac{A}{2}$, $\angle IBI_A = \angle ICI_A = 90$

(ii) B, I, C, I_A are cyclic, it's centre is the midpoint of $\widehat{BC}$ (Incentre-Excentre-Lemma)

(iii) I is the orthocentre of $\Delta I_A I_B I_C$.

(iv) (ABC) is the nine-point-circle of $\Delta I_A I_B I_C$

Problem 5. *Let D and D' be the points where Incircle and A-excircle touches BC. Let X be the diametrically opposite point to D. Show that A, X, D are collinear.*