Optimization problems

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Problem 1 (INMO 2024). *All the squares of a* 2024×2024 *board are coloured white. In one move, Mohit can select one row or column whose every square is white, choose exactly* 1000 *squares in that row or column, and colour all of them red. Find maximum number of squares Mohit can colour in a finite number of moves.*

Problem 2 (Belarus MO 2024 P10.2). *Some vertices of a regular* 2024*-gon are marked such that for any regular polygon, all of whose vertices are vertices of the* 2024*-gon, at least one of its vertices is marked. Find the minimum possible number of marked vertices.*

Problem 3 (JBMO SL 2024 C1). *Determine the smallest positive integer* k *with the following property. For any subset* $\mathbb{S}$ *of the set* $\{1, 2, 3, \cdot, 2024\}$ *with* $|S| = k$*, there are two distinct elements* $a, b \in \mathbb{S}$ *such that* $ab + 1$ *is a perfect square*

Problem 4 (Austrian MO National 2025). *Consider the following game for a positive integer* n*. Initially, the numbers* $1, 2, \ldots, n$ *are written on a board. In each move, two numbers are selected such that their difference is also present on the board. This difference is then erased from the board. For which values of* n *is it possible to end with only one number remaining on the board?*

Problem 5 (JBMO SL 2023 C1). *Given is a square board with dimensions* 2023×2023*, in which each unit cell is colored blue or red. There are exactly* 1012 *rows in which the majority of cells are blue, and exactly* 1012 *columns in which the majority of cells are red. What is the maximal possible side length of the largest monochromatic square?*

Problem 6 (JBMO SL 2021 C1). *In Mathcity, there are infinitely many buses and infinitely many stations. The stations are indexed by the powers of* $2 : 1, 2, 4, 8, 16, \ldots$ *Each bus goes by finitely many stations, and the bus number is the sum of all the stations it goes by. For simplifications, the mayor of Mathcity wishes that the bus numbers form an arithmetic progression with common difference* r *and whose first term is the favourite number of the mayor. For which positive integers* r *is it always possible that, no matter the favourite number of the mayor, given any* m *stations, there is a bus going by all of them?*

Problem 7 (JBMO SL 2023 C2). *There are* n *blocks placed on the unit squares of a* $n \times n$ *chessboard such that there is exactly one block in each row and each column. Find the maximum value* k*, in terms of* n*, such that however the blocks are arranged, we can place* k *rooks on the board without any two of them threatening each other.*

Problem 8 (Brazil MO 2025). *Let* $n \geq 4$ *be a positive integer and let* $\mathcal{P}$ *be an* n*-sided convex polygon. A set* $\mathcal{X}$ *of the diagonals of* $\mathcal{P}$ *is called blocking if any triangulation of* $\mathcal{P}$ *has at least one diagonal in* $\mathcal{X}$*. Determine, for each* n*, the least possible number of diagonals a blocking set* $\mathcal{X}$ *can have.*

Problem 9 (Al-Khwarizmi 2025 P8). *There are* 100 *cards on a table, flipped face down. Madina knows that on each card a single number is written and that the numbers are different integers from* 1 *to* 100*. In a move, Madina is allowed to choose any* 3 *cards, and she is told a number that is* *written on one of the chosen cards, but not which specific card it is on. After several moves, Madina must determine the written numbers on as many cards as possible. What is the maximum number of cards Madina can ensure to determine?*

Problem 10 (JBMO SL 2022 C1)**.** *Anna and Bob, with Anna starting first, alternately color the integers of the set* $S = \{1, 2, ..., 2022\}$ *red or blue. At their turn each one can color any uncolored number of* S *they wish with any color they wish. The game ends when all numbers of* S *get colored. Let* N *be the number of pairs* (a, b)*, where* a *and* b *are elements of* S*, such that* a*,* b *have the same color, and* $b - a = 3$*. Anna wishes to maximize* N*. What is the maximum value of* N *that she can achieve regardless of how Bob plays?*

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Problem 11 (Swiss Final Round 2023 P8)**.** *Let* n *be a positive integer. We start with* n *piles of pebbles, each initially containing a single pebble. One can perform moves of the following form: choose two piles, take an equal number of pebbles from each pile and form a new pile out of these pebbles. Find (in terms of* n*) the smallest number of nonempty piles that one can obtain by performing a finite sequence of moves of this form.*

Problem 12 (Canada MO 2024 P4)**.** *Treasure was buried in a single cell of an* $M \times N$ *(*$2 \leq M$*,* N*) grid. Detectors were brought to find the cell with the treasure. For each detector, you can set it up to scan a specific subgrid* $[a, b] \times [c, d]$ *with* $1 \leq a \leq b \leq M$ *and* $1 \leq c \leq d \leq N$*. Running the detector will tell you whether the treasure is in the region or not, though it cannot say where in the region the treasure was detected. You plan on setting up* Q *detectors, which may only be run simultaneously after all* Q *detectors are ready.*

In terms of M *and* N*, what is the minimum* Q *required to gaurantee to determine the location of the treasure?*

Problem 13 (BMO SL C1)**.** *Let* n*,* k *be positive integers. Julia and Florian play a game on a* $2n \times 2n$ *board. Julia has secretly tiled the entire board with invisible dominos. Florian now chooses* k *cells. All dominos covering at least one of these cells then turn visible. Determine the minimal value of* k *such that Florian has a strategy to always deduce the entire tiling.*

Problem 14 (India TST 2024)**.** *A sleeping rabbit lies in the interior of a convex* 2024*-gon. A hunter picks three vertices of the polygon and he lays a trap which covers the interior and the boundary of the triangular region determined by them. Determine the minimum number of times he needs to do this to guarantee that the rabbit will be trapped.*

Problem 15 (IMOSL 2016 C1)**.** *The leader of an IMO team chooses positive integers* n *and* k *with* $n > k$*, and announces them to the deputy leader and a contestant. The leader then secretly tells the deputy leader an* n*-digit binary string, and the deputy leader writes down all* n*-digit binary strings which differ from the leader's in exactly* k *positions. (For example, if* $n = 3$ *and* $k = 1$*, and if the leader chooses* 101*, the deputy leader would write down* 001*,* 111 *and* 100*.) The contestant is allowed to look at the strings written by the deputy leader and guess the leader's string. What is the minimum number of guesses (in terms of* n *and* k*) needed to guarantee the correct answer?*