Games

Amit Kumar Mallik

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Problem 1 (Classic). *A basket has n balls. Alice and Bob play the following game. Starting with Alice, each player removes either one, two or three balls from the basket. The player who cannot make a move loses. For what values of n, Alice has a winning strategy?*

Problem 2 (Rioplatense MO 2013). *Two players Alice and Bob play alternatively in a convex polygon with $n \geqslant 5$ sides. In each turn, the corresponding player has to draw a diagonal that does not cut inside the polygon previously drawn diagonals. A player loses if after his turn, one quadrilateral is formed such that its two diagonals are not drawn. A starts the game. For each positive integer n, find a winning strategy for one of the players.*

Problem 3 (Belarus MO 2025). *Polina and Yan have n cards, on the first card on one side 1 is written, on the other side $n+1$, on the second card on one side 2 is written, on the other side $n+2$, etc. Polina laid all cards in a circle in some order. Yan wants to turn some cards such that the numbers on the top sides of adjacent cards were not coprime. For every positive integer $n \geq 3$ determine can Yan accomplish that regardless of the actions of Polina.*

Problem 4 (Argentina MO 2020 P6). *Let $n \geqslant 3$ be an integer. Lucas and Matias play a game in a regular n-sided polygon with a vertex marked as a trap. Initially Matias places a token at one vertex of the polygon. In each step, Lucas says a positive integer and Matias moves the token that number of vertices clockwise or counterclockwise, at his choice.*

(i) Determine all the $n \geq 3$ such that Matias can locate the token and move it in such a way as to never fall into the trap, regardless of the numbers Lucas says. Give the strategy to Matias.

(ii) Determine all the $n \geq 3$ such that Lucas can force Matias to fall into the trap. Give the strategy to Lucas.

Problem 5 (Game of Chomp). *There is a chocolate bar of size $m \times n$. Alice and Bob eat the chocolate alternately, starting with Alice. On their turn, they must choose a (1×1) block and eat it, together with all the blocks that are below it and to its right. The top left block is poisoned.*

(i) For $m = n$, find a strategy for one of them to survive.

(ii) For what values of (m, n), Alice has a winning strategy. Can you find such a winning strategy.

Problem 6 (Argentina MO 2024). *On a table, there are $10\,000$ matches, two of which are inside a box. Ana and Beto take turns playing the following game. On each turn, a player adds to the box a number of matches equal to a proper divisor of the current number of matches in the box. The game ends when, for the first time, there are more than 2024*

matches in the box and the person who played the last turn is the winner. If Ana starts the game, determine who has a winning strategy

Problem 7 (ELMO SL 2019 C1)**.** *Elmo and Elmo's clone are playing a game. Initially, $n \geq 3$ points are given on a circle. On a player's turn, that player must draw a triangle using three unused points as vertices, without creating any crossing edges. The first player who cannot move loses. If Elmo's clone goes first and players alternate turns, who wins? (Your answer may be in terms of n.)*

Problem 8 (USAMO 2023 P4)**.** *A positive integer a is selected, and some positive integers are written on a board. Alice and Bob play the following game. On Alice's turn, she must replace some integer n on the board with $n+a$, and on Bob's turn he must replace some even integer n on the board with $n/2$. Alice goes first and they alternate turns. If on his turn Bob has no valid moves, the game ends.*

After analyzing the integers on the board, Bob realizes that, regardless of what moves Alice makes, he will be able to force the game to end eventually. Show that, in fact, for this value of a and these integers on the board, the game is guaranteed to end regardless of Alice's or Bob's moves.

Problem 9 (JBMO P3)**.** *Alice and Bob play the following game on a 100×100 grid, taking turns, with Alice starting first. Initially the grid is empty. At their turn, they choose an integer from 1 to 100^2 that is not written yet in any of the cells and choose an empty cell, and place it in the chosen cell. When there is no empty cell left, Alice computes the sum of the numbers in each row, and her score is the maximum of these 100 numbers. Bob computes the sum of the numbers in each column, and his score is the maximum of these 100 numbers. Alice wins if her score is greater than Bob's score, Bob wins if his score is greater than Alice's score, otherwise no one wins.*

Find if one of the players has a winning strategy, and if so which player has a winning strategy.

Problem 10 (Austria 2021 National)**.** *On a blackboard, there are 17 integers not divisible by 17. Alice and Bob play a game. Alice starts and they alternately play the following moves: • Alice chooses a number a on the blackboard and replaces it with a^2 • Bob chooses a number b on the blackboard and replaces it with b^3. Alice wins if the sum of the numbers on the blackboard is a multiple of 17 after a finite number of steps. Prove that Alice has a winning strategy.*

Problem 11 (Brazil MO 2025)**.** *Ana and Banana are going to play tic-tac-toe, but both already know the winning strategy, so it always ends in a draw! With that in mind, Ana suggests a new game—one that never draws—the Super Tic-Tac-Toe! In this game, Ana and Banana will play alternately on an $n \times n$ board, initially with 0 in every cell. Each move consists of choosing a cell of the board and adding an integer from 1 to c to it, so that no cell exceeds the number m. The winner is whoever makes a move that completes a row, a column, or one of the two diagonals with the number m in every cell of it. For which triples of positive integers (n, m, c) does Ana, who plays first, have a winning strategy?*

Problem 12 (Iran MO 2024 P2)**.** *Sahand and Gholam play on a 1403×1403 table. Initially* *all the unit square cells are white. For each row and column there is a key for it (totally 2806 keys). Starting with Sahand players take turn alternatively pushing a button that has not been pushed yet, until all the buttons are pushed. When Sahand pushes a button all the cells of that row or column become black, regardless of the previous colors. When Gholam pushes a button all the cells of that row or column become red, regardless of the previous colors. Finally, Gholam's score equals the number of red squares minus the number of black squares and Sahand's score equals the number of black squares minus the number of red squares. Determine the minimum number of scores Gholam can guarantee without if both players play their best moves.*

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Problem 13 (Balkan MO Shortlist 2023 C1)**.** *Joe and Penny play a game. Initially there are* 5000 *stones in a pile, and the two players remove stones from the pile by making a sequence of moves. On the* k*-th move, any number of stones between* 1 *and* k *inclusive may be removed. Joe makes the odd-numbered moves and Penny makes the even-numbered moves. The player who removes the very last stone is the winner. Who wins if both players play perfectly?*

Problem 14 (Canada MO 2019 P5)**.** *A 2-player game is played on* $n \geq 3$ *points, where no 3 points are collinear. Each move consists of selecting 2 of the points and drawing a new line segment connecting them. The first player to draw a line segment that creates an odd cycle loses. (An odd cycle must have all its vertices among the* n *points from the start, so the vertices of the cycle cannot be the intersections of the lines drawn.) Find all* n *such that the player to move first wins.*

Problem 15 (APMO 2022 P4)**.** *Let* n *and* k *be positive integers. Cathy is playing the following game. There are* n *marbles and* k *boxes, with the marbles labelled* 1 *to* n*. Initially, all marbles are placed inside one box. Each turn, Cathy chooses a box and then moves the marbles with the smallest label, say* i*, to either any empty box or the box containing marble* $i + 1$*. Cathy wins if at any point there is a box containing only marble* n*. Determine all pairs of integers* (n, k) *such that Cathy can win this game.*