INMOTC 2026 (MP region)

ALGEBRA

2 January 2026

List of problems and examples

Contents

§1 Problems

Exercise 1.1 (USAMO 1975 P3, AoPS)**.** A polynomial $P(x)$ of degree n satisfies

$$P(k) = \frac{k}{k+1} \qquad \text{for } k = 0, 1, 2, \ldots, n.$$

Find $P(n+1)$.

Walkthrough —

(a) Consider the polynomial $(x+1)P(x) - x$.

Example 1.2. Let $P(x)$ be a polynomial with real coefficients such that $P(\sin\alpha) = P(\cos\alpha)$ for all $\alpha \in \mathbb{R}$. Show that $P(x) = Q(x^2 - x^4)$ for some polynomial $Q(x)$ with real coefficients.

Exercise 1.3 (International Mathematics Competition 2008 Day 2 P4)**.** Let $f(x), g(x)$ be nonconstant polynomials with integer coefficients such that $g(x)$ divides $f(x)$. Prove that if the polynomial $f(x) - 2008$ has at least 81 distinct integer roots, then the degree of $g(x)$ is greater than 5.

> **Walkthrough** —
>
> **(a)**

Exercise 1.4 (All-Russian Mathematical Olympiad 2007 Grade 11 Day 2 P6, AoPS, by N. Agakhanov, I. Bogdanov)**.** Do there exist nonzero reals a, b, c such that, for any $n > 3$, there exists a polynomial $P_n(x) = x^n + \cdots + ax^2 + bx + c$, which has exactly n (not necessarily distinct) integral roots?

> **Walkthrough** —
>
> **(a)**

Example 1.5. Does there exist a polynomial $P(x)$ with rational coefficients such that $\sin x = P(x)$ for all $x \geq 100$?

Example 1.6. Let $f(x), g(x)$ be polynomials with integer coefficients. Assume that for infinitely many integers n, $g(n)$ is nonzero and divides $f(n)$. Show that $g(x)$ divides $f(x)$ in the ring of polynomials with rational coefficients.

Example 1.7. Let n be a positive integer. Show that the polynomial

$$(x-1)(x-2)\cdots(x-n)-1$$

cannot be factored into the product of two non-constant polynomials with integer coefficients. What can be said about the polynomial

$$(x-1)(x-2)\cdots(x-n)+1?$$