INMO 2016

Exercise 1 (INMO 2016 P4,). Suppose 2016 points of the circumference of a circle are coloured red and the remaining points are coloured blue. Given any natural number $n \geq 3$, prove that there is a regular n-sided polygon all of whose vertices are blue.

Solution 1. Draw a regular n-gon circumscribed in the circle. If all its vertices are blue, then we are done. Otherwise, we rotate the regular n-gon by a sufficiently small angle so that all its vertices become red. This is possible since there are only finitely many blue points on the circle. ■