INMO 2001

Exercise 1 (INMO 2001 P3,). If a, b, c are positive real numbers such that $abc = 1$, Prove that

$$a^{b+c}b^{c+a}c^{a+b} \leq 1.$$

Solution 1. Put

$$\alpha = a^{b+c}b^{c+a}c^{a+b},$$
$$\beta = b^{b+c}c^{c+a}a^{a+b},$$
$$\gamma = c^{b+c}a^{c+a}b^{a+b}.$$

Note that $\alpha\beta\gamma = 1$, and by the rearrangement inequality, it follows that

$$\alpha \leq \beta, \alpha \leq \gamma$$

hold. This gives $\alpha^3 \leq 1$, which yields $\alpha \leq 1$. ■

> **Remark.** The above proof is similar to be the proof of Nesbitt's inequality as in [Hun08].

References

[**Hun08**] Pham Kim Hung. *Secrets in Inequalities. Volume 1: basic inequalities*. Gil Publishing House, 2008 (cited p. 1)